II. Curve $y^2(2a - x) = x^3$

(i) By putting $x = 0, y = 0$ in the equation of the curve, both the sides of the curve are satisfied, therefore the curve passes through the origin.

(ii) Since the equation of the curve contains only even powers of y therefore the curve is symmetrical about x-axis.

(iii) By putting $y = 0$ in the equation of the curve we find that $x = 0$. Similarly by putting $x = 0$ we find that $y = 0$. This means that the curve does not cut the co-ordinate axes at any other point except at the origin.

(iv) By equating to zero the term of the lowest degree from the equation of the curve, viz. $2ay^2 = 0$, we find that $y = 0$. That is, x-axis ($y = 0$), is the tangent to the curve at the origin.

(v) By equating to zero the coefficient of the term of the highest degree in y we find $(2a - x) = 0$ i.e. $x = 2a$. Therefore the asymptote of the curve parallel to y-axis is $x = 2a$.

(vi) Again, solving for y from the equation of the curve, we get

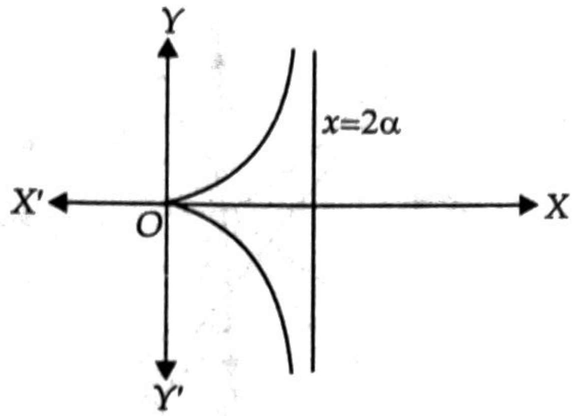
$$y^2 = \frac{x^3}{2a - x}.$$

Now if x is $-ve$, then $y^2 = -ve$ $\therefore$ y is imaginary

and if $x > 2a$, then $y^2 = -ve$ $\therefore$ y is imaginary.

Therefore there will be no portion of the curve on the L.H.S. of $x = 0$ or on the R.H.S. of $x = 2a$.

Hence the graph of the curve will be as follows :



III. Curve $x^3 + y^3 = 3axy$ (Folium of Descartes)

(i) Since there is no constant term in the equation of the curve, therefore the curve passes through the origin.

(ii) If we put y for x and x for y in the equation of the curve, it does not change. This means that the curve is symmetrical about the line $y = x$.

(iii) By equating to zero the term of the lowest degree in the equation of the curve we get $3axy = 0$ which $\Rightarrow x = 0, y = 0$ i.e. the equations of the tangents at the origin are $x = 0$ and $y = 0$.

Thus origin is a double point.

(iv) By putting $y = 0$ in the equation of the curve we get $x = 0$ and similarly by putting $x = 0$ we get $y = 0$. Hence the curve cuts both the co-ordinate axes only at the origin.

(v) x and y cannot be negative simultaneously, for in that case L.H.S. = $-ve$ and R.H.S. = $+ve$.

Thus there will be no portion of the curve in the third quadrant (where $x = -ve$ and $y = -ve$).

(vi) The only real asymptote of the curve is $x + y + a = 0$ (this has been derived in the chapter on asymptote in page 331 of Diff. Calculus).

(vii) This curve cuts the line $y = x$ in those points where

$$x^3 + y^3 = 3axy.$$

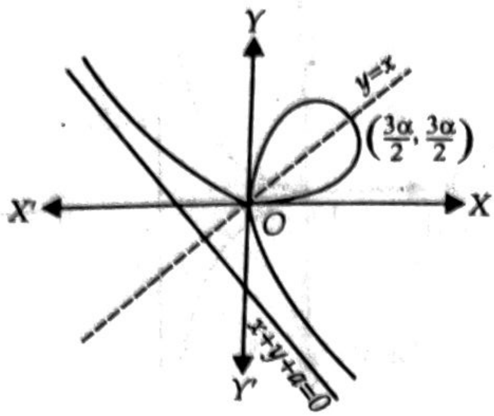
This $\Rightarrow$ $x^3 + x^3 = 3ax \cdot x \Rightarrow 2x^3 = 3ax^2$

$\Rightarrow$ $x^2(2x - 3a) = 0, \quad \therefore \; x = 0, x = \dfrac{3a}{2}.$

That is, the curve cuts the line $y = x$ in two points

$$(0, 0) \text{ and } \left(\frac{3a}{2}, \frac{3a}{2}\right).$$

Hence the graph of the curve is as follows :



IV. Curve $y^2(a-x) = x^2(a+x)$.

(i) By putting $x = 0$, $y = 0$ in the equation of the curve both the sides are satisfied. Hence the curve passes through the origin.

(ii) Since the index of y is even, therefore the curve is symmetrical about the x-axis.

(iii) By putting $y = 0$ in the equation of the curve, we find that $x^2(a+x) = 0 \therefore x = 0$, and $x = -a$, i.e. the curve cuts the x-axis at the point $(-a, 0)$.

Again putting $x = 0$ we get $y = 0$ i.e. the curve cuts the y-axis at the point $(0, 0)$.

(iv) From the equation of the given curve, we get

$$y^2 = \frac{x^2(a+x)}{a-x}$$

$$\Rightarrow \qquad y = x\sqrt{\frac{a+x}{a-x}}.$$

Now if in this equation we give to x, values such that $x < -a$ or $x > a$ then y becomes imaginary. Therefore there will be no portion of the curve on the L.H.S. of $x = -a$ or on the R.H.S. of $x = a$.

(v) By equating to zero the term of the lowest degree in the equation of the curve we get $y^2 - x^2 = 0$ i.e. $y = \pm x$.

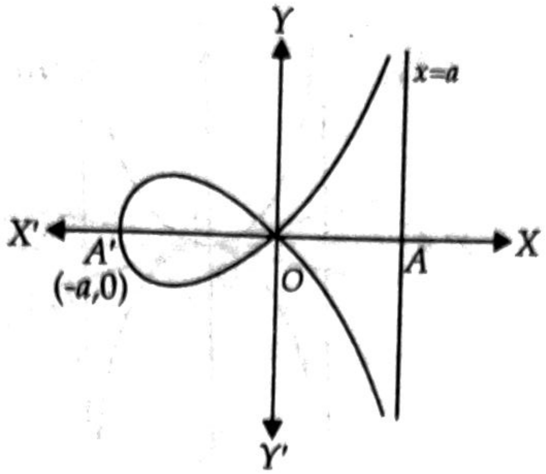
That is, the equation of the tangent at the origin is $y = \pm x$.

(vi) By equating to zero the coefficient of the term containing the highest power of y we get $a - x = 0$.

Hence the equation of the asymptote parallel to y-axis is $x = a$.

(vii) At the point $(-a, 0)$ we have $\frac{dy}{dx} = \infty$, i.e. at the point $(-a, 0)$, the tangent will be perpendicular to x-axis.

Hence the graph of the curve will be as follows :



V. Curve $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

The given curve can be written in the following form :

$$a^2y^2 - b^2x^2 = x^2y^2.$$

(i) Since putting $x = 0$, $y = 0$, both the sides of the equation are satisfied, therefore the curve passes through the origin.

(ii) Since the indices of x and y both are even, therefore the curve is symmetrical about both the axes.

(iii) Putting $y = 0$ in the equation of the curve, we get $x = 0$. Similarly putting $x = 0$ we get $y = 0$. This means that the curve does not cut the axes elsewhere except at the origin.

(iv) By equating to zero the term of the lowest degree in the equation of the curve we get $a^2y^2 - b^2x^2 = 0$.

This $\Rightarrow$ $ay = \pm bx$. That is, the equation of the tangent at the origin are $ay = \pm bx$.

(v) By equating to zero, the coefficient of the term containing the highest degree in y in the equation of the curve, we have $x^2 - a^2 = 0$. This $\Rightarrow$ $x = \pm a$.

Therefore the equations of the asymptotes parallel to the y-axis are $x = \pm a$.

Similarly by equating to zero, the term containing the highest degree in y, we have $b^2 + y^2 = 0$ from which y is imaginary. That is, the asymptotes parallel to the x-axis are imaginary.

(vi) Solving for y from the equation of the curve, we get

$$y^2(a^2 - x^2) = b^2x^2 \; \therefore \; y = \frac{bx}{\sqrt{a^2 - x^2}}$$

Now, if we put $x > a$ or $x < -a$ in this equation, then y becomes imaginary. Therefore there is no portion of the curve outside $x = a$ or $x = -a$.

Hence the graph of the curve will be as follows :